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Quantum Switching Effect on Vacuum Fluctuations

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We investigate properties of the quantum vacuum fluctuations in a space with boundaries. We here focus on the quantum fluctuations of the electromagnetic vacuum in a half-space bounded by a perfectly reflecting flat mirror. We in particular study the measurement process of the vacuum fluctuations through the quantum Brownian motion, i.e. the velocity dispersion of a charged probe particle released in the vacuum. Two important new phenomena regarding the vacuum measurement are reported here. One is the switching effect and the other is the smearing effect.

The first phenomenon to be investigated is the switching effect. Constructing a smooth switching function by gluing together a plateau and the Lorentzian switching tails, it is shown that the switching tails have a great influence on the measurement of the Brownian motion in the quantum vacuum. Indeed the result with a smooth switching function and the one with a sudden switching function are qualitatively quite different. It turns out that the anti-correlations between the main measuring part and the switching tails plays an essential role in this switching effect.

The second phenomenon to be discussed is the smearing effect. We find out that the spread of the probe particle significantly influences the measured velocity dispersion, and the latter so obtained shows reasonable late-time behavior. In particular the asymptotic behavior of the z-component, $\langle \Delta v_z^2 \rangle$, is $\langle \Delta v_z^2 \rangle \sim 1/\tau^2$ as $\tau \to \infty$ (τ is the measuring time). It is interpreted that anti-correlations of the fluctuations between the center-part and the tail-part of the wave-packet are essential elements for yielding this behavior. The present results not only resolves the previously reported puzzle of the peculiar late-time behavior $\langle \Delta v_z^2 \rangle \sim 1/z^2$ for a point-particle probe (z is the distance from the mirror-boundary to the particle), but also can be quite significant for investigating various related problems.

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I. INTRODUCTION

Quantum vacuum often plays a key role in various situations related to quantum physics. One of the significant features of quantum vacuum is that, though it usually appears in the context of microscopic physics, the vacuum itself is essentially a global concept and the spectral profile of the vacuum sensitively reflects global conditions of the corresponding field. Furthermore, the averaged net quantity in the vacuum is basically zero, so that the subtle effects reflecting global conditions become dominant and detectable in some cases. The Casimir effect may be the most well-known example illustrating these features of quantum vacuum. In this sense, quantum vacuum bridges the gap between the microscopic concept and the macroscopic concept, and deeper understanding of quantum physics may be acquired when we get more insights on quantum vacuum.

One convenient way of studying vacuum fluctuations is to analyze the quantum Brownian motion of a test particle released in the quantum vacuum in question. Then the velocity dispersion of the probe particle is an appropriate quantity to investigate.

In this report, we study two new effects regarding the measuring process of the vacuum fluctuations near a perfectly reflecting mirror-boundary. They are the *switching effect* and the *smearing effect*. Detailed treatments are found in Ref.'s [2] and [4].

Starting with the model discussed by Yu and Ford [1], we shall first investigate the *switching effect* in the measurement process by constructing a convenient switching function of time, consisting of a main measuring plateau with two Lorentzian tails. The switching process can be regarded as a non-trivial, time-dependent interactions between the probe particle and the vacuum fluctuations and it sheds some light on hidden properties of the quantum vacuum. We find out that the measurement of the quantum vacuum fluctuations is, contrary to the usual macroscopic measurements, drastically influenced by the switching tails and that the anti-correlation between the main measuring part and the switching tails is playing an essential role in the process. Here the important factor is the non-stationary interactions between the probe and the violent fluctuations of the vacuum.

We shall next investigate the *smearing effect* caused by the quantum spread of the probe particle. In reality a probe particle is also a quantum object and should follow the quantum principle. As the first step in this direction, then, we here treat a probe particle as a Gaussian wave-packet. Then the spread of the probe particle gives rise to the smearing of the time-correlation functions. What we shall find out is that the late-time behavior of $\langle \Delta v_z^2 \rangle$ turns out to be $\langle \Delta v_z^2 \rangle \sim 1/\tau^2$ (τ is the measuring time) rather than the previously reported be-

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havior [1] $\langle \Delta v_z^2 \rangle \sim 1/z^2$, *as far as the spread of the probe particle is taken into account.* It shall be found out that the point-particle case corresponds to a measure-zero set in the whole set to be considered for a spread-probe case. Thus provided that the probe-particle is treated as spreading, the measure-zero effect does not appear in the final result. On the other hand, when one starts with a point-particle probe from the out set (as so in Ref.[1]), only the contribution which would otherwise be regarded as measure-zero shows up, yielding the peculiar late-time behavior of $\langle \Delta v_z^2 \rangle$. Since it is more natural to regard the probe a spreading particle (due to quantum effect) instead of a point-particle, one would see that the present result is more feasible.

Though based on a simple model, our results can be quite significant since there have been many arguments so far based on the sudden-switching assumption and/or the point-particle assumption.

II. MIRROR-BOUNDARY MODEL FOR VACUUM FLUCTUATIONS

The model we investigate is based on the one discussed in Ref.[1] and reanalyzed in Ref.[2].

Preparing a flat, infinitely spreading mirror of perfect reflectivity placed on the xy-plane ($z = 0$), the quantum vacuum of the electromagnetic field is considered inside the half-space $z > 0$. Let the quantum fluctuations of the vacuum be probed by a classical charged particle with mass m and charge e. When the particle velocity is much smaller than c, the particle couples only with the electric field $\vec{E}(\vec{x}, t)$, and the motion for the particle is described by

$$m\frac{d\vec{v}}{dt} = e\vec{E}(\vec{x}, t) \quad . \tag{1}$$

Assuming that the position of the particle does not change so much within the time-scale in question, Eq. (1) along with the initial condition $\vec{v}(0) = \vec{v}_0$ is approximately solved to

$$\vec{v}(t) \simeq \vec{v}_0 + \frac{e}{m}\int_0^t \vec{E}(\vec{x}, t')dt' \quad . \tag{2}$$

In the "sudden-switching" case discussed in Ref.[1], the measurement is a step-function-like process characterized by abrupt switching on/off without no switching tails. It is described by a step-like switching function

$$\begin{aligned} \Theta(t) &= 1 \;\; (\text{for } \; 0 \le t \le \tau) \\ &= 0 \;\; (\text{otherwise}) \quad . \end{aligned} \tag{3}$$

In this case the velocity dispersions of the particle, $\langle \Delta v_i{}^2 \rangle$ ($i = x, y, z$), are given by

$$\langle \Delta v_i{}^2(\vec{x}, \tau)\rangle = \frac{e^2}{m^2}\int_0^\tau dt' \int_0^\tau dt'' \langle E_i(\vec{x}, t')E_i(\vec{x}, t'')\rangle_R \quad , \tag{4}$$

where $\langle E_i(\vec{x}, t')E_i(\vec{x}, t'')\rangle_R$ ($i = x, y, z$) are the renormalized two-point correlation functions ("R" is for "renormalized"). Here we note $\langle E_i(\vec{x}, t)\rangle_R = 0$. Explicit expressions for $\langle E_i(\vec{x}, t')E_i(\vec{x}, t'')\rangle_R$ ($i = x, y, z$) are known[3] as

$$\langle E_z(\vec{x}, t')E_z(\vec{x}, t'')\rangle_R = \frac{1}{\pi^2}\frac{1}{(T^2 - (2z)^2)^2} \tag{5}$$

$$\begin{aligned} \langle E_x(\vec{x}, t')E_x(\vec{x}, t'')\rangle_R &= \langle E_y(\vec{x}, t')E_y(\vec{x}, t'')\rangle_R \\ &= -\frac{1}{\pi^2}\frac{T^2 + 4z^2}{(T^2 - (2z)^2)^3} \quad , \end{aligned} \tag{6}$$

where $T := t' - t''$. (We set $c = \hbar = 1$ hereafter throughout the paper.)

Now we generalize the above original model into two directions. One direction of generalization is to replace Eq.(3) with a more natural switching function. We then find out a *switching effect* [2] in the vacuum measuring process. The other direction of generalization is to take into account the quantum spread of the probe particle and to investigate the *smearing effect* [4].

Let us first note that, in general, velocity dispersions are estimated by the integral of the form

$$\mathcal{I} = \int_{-\infty}^{\infty} dt' \int_{-\infty}^{\infty} dt'' \; F(t')F(t'')\mathcal{K}(t' - t'') \quad , \tag{7}$$

where the function $F(t)$ is an appropriate switching function mimicking an actual measurement process; the integral kernel $\mathcal{K}$ is assumed to be an even function of $T := t' - t''$ with an appropriate asymptotic behavior as $|T| \to \infty$. When we choose $F(t)$ appropriately, then we can discuss the switching effect, while choosing a suitable smeared kernel as $\mathcal{K}(T)$, we can analyze the smearing effect. We shall briefly sketch these two investigations one by one below. (See Ref.[2] and Ref.[4] for detailed treatments.)

III. SWITCHING EFFECT

We construct a "Lorentz-plateau" function $F_{\tau\mu}(t)$ [2], which is defined by

$$\begin{aligned} F_{\tau\mu}(t) &= 1 \qquad\qquad\qquad\qquad (\text{for } \; |t| \le \tau/2) \\ &= \frac{\mu^2}{(|t|/\tau - 1/2)^2 + \mu^2} \;\; (\text{for } \; |t| > \tau/2) \end{aligned} \tag{8}$$

where τ and μ are positive parameters. Its plateau part corresponds to the stable measuring period, while two Lorentzian tails corresponding to switching-tails. The time-scale for the measuring-part is $\tau_1 := \tau$, while the time-scale of the switching-tails is characterized by $\tau_2 := 2\int_{\tau/2}^{\infty} F_{\tau\mu}(t)dt = \pi\mu\tau$. The dimension-free parameter $\mu = \frac{\tau_2}{\pi\tau_1}$ is then the switching-duration parameter, describing the relative switching duration compared to the main measuring time-scale.

Plugging Eq.(8) and Eq.(5) into $F(t)$ and $\mathcal{K}$ in Eq.(7), respectively, we get [2]

$$
\begin{aligned}
\langle \Delta v_z{}^2 \rangle &= \frac{2\, e^2}{\pi^2 m^2 \tau^2} \int_0^1 dx\, \frac{1-x}{(x^2-\sigma_1{}^2)^2} \\
&+ \frac{4\, e^2}{\mu^2 \pi m^2 \tau^2} \int_0^\infty d\chi\, \frac{1}{(\chi^2+4)(\chi^2-\sigma_2{}^2)^2} \\
&+ \frac{4\, e^2}{\mu^2 \pi^2 m^2 \tau^2} \int_0^\infty d\chi\, \times \\
&\times \left\{ \frac{1}{(\chi^2-\sigma_2{}^2)^2} - \frac{1}{\{(\chi+1/\mu)^2-\sigma_2{}^2\}^2} \right\} \mathcal{F}(\chi) \\
&=: \langle \Delta v_z{}^2 \rangle_M + \langle \Delta v_z{}^2 \rangle_S + \langle \Delta v_z{}^2 \rangle_{MS} \ , \qquad (9)
\end{aligned}
$$

where $\mathcal{F}(\chi)$ is given by

$$
\mathcal{F}(\chi) := \left(1 - \frac{1}{\chi^2+4}\right) \tan^{-1}\chi - \frac{1}{\chi(\chi^2+4)} \ln(1+\chi^2) \ . \qquad (10)
$$

Here we have introduced dimensionless parameters,

$$
\sigma_1 := 2z/\tau \ , \quad \sigma_2 := \sigma_1/\mu \ , \quad \chi := x/\mu \ . \qquad (11)
$$

Here the suffixes "M" and "S" are for "measuring" and "switching", respectively. The term $\langle \Delta v_z{}^2 \rangle_M$ comes from the two-point correlation solely within the measuring part ($|t| < \tau/2$) and coincides with the one obtained in Ref.[1] for the sudden-switching case. The term $\langle \Delta v_z{}^2 \rangle_S$ originates from switching tails, while the term $\langle \Delta v_z{}^2 \rangle_{MS}$ from the correlation between the measuring part and the switching tail.

Here we just mention that computing the singular integrals such as those in Eq.(9) requires some regularization method [2] and that we have here resorted to the generalized principal-value method [5].

One thing to be noted is the term $\langle \Delta v_z{}^2 \rangle_{MS}$ which is estimated as

$$
\begin{aligned}
\langle \Delta v_z{}^2 \rangle_{MS} &= O(1) \cdot \frac{2e^2}{\mu^2 \pi m^2 \tau^2} \times \int_0^{1/\mu} \frac{1}{(\chi^2-\sigma_2{}^2)^2} d\chi \\
&\approx -O(1) \cdot \frac{2\mu e^2}{3\pi m^2 \tau^2} \\
&\sim -O(\mu \sigma_1^2) \langle \Delta v_z{}^2 \rangle_M \ . \qquad (12)
\end{aligned}
$$

The origin of this *anti-correlation* effect resides in the correlation between the measuring part and the switching tail. In this way, it is seen that the interplay between the measuring part and the switching tail is a key to understand the measurement process of quantum vacuum.

In terms of the three parameters τ_1, τ_2 and z, one can classify the late-time behavior of $\langle \Delta v_z{}^2 \rangle$ into the following four types. (We naturally assume $\tau_1 \gg 2z$ for these late-time behaviors.)

(i) When $\tau_2 \ll 2z \ll \tau_1$, $\langle \Delta v_z{}^2 \rangle \approx \langle \Delta v_z{}^2 \rangle_M$.

(ii) When $\tau_2 \approx 2z \ll \tau_1$, $\langle \Delta v_z{}^2 \rangle \approx \frac{3}{2} \langle \Delta v_z{}^2 \rangle_M$.

(iii) When $2z \ll \tau_1 \ll \tau_2$ and $\frac{\tau_2}{\tau_1} = O\left(\left(\frac{\tau_1}{2z}\right)^2\right)$,

$$\langle \Delta v_z{}^2 \rangle \approx \langle \Delta v_z{}^2 \rangle_S .$$

(iv) When $2z \ll \tau_1 \ll \tau_2$ and $\frac{\tau_2}{\tau_1} \gg \frac{\tau_1^2}{(2z)^2}$,

$$
\begin{aligned}
\langle \Delta v_z{}^2 \rangle &\approx -O\left(\frac{\tau_2}{\tau_1}\left(\frac{2z}{\tau_1}\right)^2\right) \cdot \langle \Delta v_z{}^2 \rangle_M \\
&\sim -O(1) \cdot \frac{2\, e^2}{3 m^2 \pi^2} \frac{\tau_2}{\tau_1^3} .
\end{aligned}
$$

When the time-scale τ_2 of the switching tails is much shorter than the time-scale $2z$, the velocity dispersion $\langle \Delta v_z{}^2 \rangle$ reduces to the result of the sudden switching case given in Ref.[1] (the case (i)). As the time-scale τ_2 increases up to around the time scale $2z$, however, $\langle \Delta v_z{}^2 \rangle$ becomes around 3/2 times of $\langle \Delta v_z{}^2 \rangle_M$ (the case (ii)). It means that the contribution from the switching tails, $\langle \Delta v_z{}^2 \rangle_S$, is almost of the same order as the contribution from the measuring part, $\langle \Delta v_z{}^2 \rangle_M$. Hence the condition for the switching to be regarded as the "sudden switching" is $\tau_2 \ll 2z$, i.e. the switching time-scale is much smaller than the scale characterizing the system configuration.

Next, as the switching time τ_2 increases the velocity dispersion decreases, reducing to the Lorentzian switching case [2] at around $\tau_2 \sim O\left(\left(\frac{\tau_1}{2z}\right)^2\right) \tau_1$ (the case (iii)). This occurs mainly due to the cancellation of the M-term by the negative contribution from the MS-term, which is actually the correlation between the switching part and the main measuring part.

Finally, the case (iv) shows the possible total negative dispersion when the switching time is really large.

It should be noted that there is an essential difference between the measurement in the quantum vacuum and the usual macroscopic measurement. In the usual measurement process, the object to be measured is more or less macroscopic so that the contributions from the switching tails are negligible compared to the main measured quantity. In the vacuum case, however, the expectation values of quantities are basically zero so that the contributions from the switching tails become the dominant part of the measurement. Thus we should not naively treat the vacuum measurement in the same manner as the usual measurement. There have been many naive arguments regarding the measurement in quantum vacuum without sufficient care on this point. Thus it is important to reanalyze these arguments taking into account the switching effect found here.

IV. SMEARING EFFECT DUE TO THE SPREAD OF THE PROBE PARTICLE

Let us now study the *smearing effect* due to the quantum nature of the probe particle. In reality the probe particle is also a quantum object and it cannot escape from the uncertainty principle. The quantum spread of the probe particle, thus, should cause the smearing effect in the measurement process of the vacuum fluctuations. Though it might be most desirable, treating the whole system (the field, the mirror and the probe particle) as a fully quantum system is not very practical. Here in this paper, then, we describe the spread of the probe by a Gaussian wave-packet. Now a typical smearing function is given by a function of the form

$$g(s-t) = \frac{1}{\sqrt{2\pi}b}\exp(-\frac{1}{2b^2}(s-t)^2) \ , \qquad (13)$$

which is the Gaussian distribution of s around the peak t with width b.

We introduce several variables and parameters for later convenience: Let τ be a time-scale characterizing the main measuring process as in the previous section. Furthermore, we introduce

$$\begin{aligned} &T := t' - t'' \ , \ S := s' - s'' \ , \\ &\nu := (S-T)/\tau \ , \ \beta := \sqrt{2}\, b/\tau \ , \\ &\sigma := 2z/\tau \ . \end{aligned} \qquad (14)$$

It is now straightforward to show that the smeared kernel becomes

$$\begin{aligned} \hat{\mathcal{K}}(T,\beta) &= \int_{-\infty}^{\infty} ds' \int_{-\infty}^{\infty} ds''\, g(s'-t')\, g(s''-t'')\, \mathcal{K}(S) \\ &= \mathcal{G}_\nu(\beta)\,[\mathcal{K}(T+\tau\nu)] \ , \end{aligned} \qquad (15)$$

where "$\mathcal{G}_\nu(\beta)[\cdot]$" in the last line indicates the Gaussian integral-transformation with respect to ν with the root-mean-square β; it is defined as

$$\mathcal{G}_\nu(\beta)\,[f(\nu)] := \frac{1}{\sqrt{2\pi}\beta}\int_{-\infty}^{\infty} d\nu\, e^{-\frac{\nu^2}{2\beta^2}} f(\nu) \qquad (16)$$

for any function $f(\nu)$. We note that $\frac{1}{\sqrt{2\pi}\beta}e^{-\frac{\nu^2}{2\beta^2}} \to \delta(\nu)$ as $\beta \to 0$, so that

$$\lim_{\beta\to 0} \mathcal{G}_\nu(\beta)\,[f(\nu)] = f(0) \ . \qquad (17)$$

Thus $\hat{\mathcal{K}}(T,\beta) \to \mathcal{K}(T)$ as $\beta \to 0$, recovering the original kernel for the point-particle limit. In view of Eq.(7), then, what we need to investigate is

$$\hat{\mathcal{I}} = \int_{-\infty}^{\infty} dt' \int_{-\infty}^{\infty} dt''\ F(t')F(t'')\hat{\mathcal{K}}(T,\beta) \ . \qquad (18)$$

Due to linearity of the integral-transformation $\mathcal{G}_\nu(\beta)[\cdot]$, one can also represent $\hat{\mathcal{I}}$ as

$$\hat{\mathcal{I}} = \mathcal{G}_\nu(\beta)\,[\mathcal{I}(\nu)] \ , \qquad (19)$$

where

$$\mathcal{I}(\nu) = \int_{-\infty}^{\infty} dt' \int_{-\infty}^{\infty} dt''\ F(t')F(t'')\mathcal{K}(T+\tau\nu) \ . \qquad (20)$$

Equations (19) along with (20) serve as general formulas for Gaussian smearing needed in our analysis.

Let us note that $\mathcal{I}(\nu)$ is an even function of ν as far as the kernel $\mathcal{K}(T)$ is an even-function of T, irrespective of the form of $F(t)$. Note also that, as $\beta \to 0$, $\hat{\mathcal{I}}$ reduces to $\mathcal{I}(\nu=0)$, the original integral without smearing (Eq.(7)).

From now on, we focus on the "measuring part" which corresponds to the first term $\langle \Delta {v_z}^2\rangle_M$ in Eq.(9) (recall that the suffix "M" is for "measuring part"),

$$\hat{\mathcal{I}}_M := \mathcal{G}_\nu(\beta)\,[\mathcal{I}_M(\nu)] \qquad (21)$$

with

$$\mathcal{I}_M(\nu) = 2\tau^2 \int_0^1 d\xi\ (1-\xi)\,\{\mathcal{K}(\tau(\xi+\nu))\}_{S(\nu)} \, . \qquad (22)$$

Here we have introduced a symmetrization symbol $\{\ \}_S$ defined as

$$\{f(u)\}_{S(u)} := \frac{1}{2}\,(f(u)+f(-u)) \qquad (23)$$

for any function $f(u)$.

The integral $\hat{\mathcal{I}}_M$ is what should be compared with the result of the original analysis of Ref. [1].

Now, plugging Eq.(5) into $\mathcal{K}$ in Eq.(22), one can compute the velocity dispersion in the z-direction. The result is [4]

$$\begin{aligned} \langle \Delta v_z^2\rangle_M = &\frac{4e^2}{\pi^2 m^2 \tau^2 \sigma^2} \times \\ &\times \mathcal{G}_\nu(\beta)\,[\mathcal{Z}(\sigma,1+\nu) - \mathcal{Z}(\sigma,\nu)] \end{aligned} \qquad (24)$$

where

$$\mathcal{Z}(\sigma,\nu) \ = \ \frac{|\nu|}{16\sigma}\ln\left(\frac{|\nu|+\sigma}{|\nu|-\sigma}\right)^2 \ , \qquad (25)$$

which is an even-function of $\sigma := 2z/\tau$. Here the generalized principal-value method [5] for singular integrals has been applied to reach the above result.

We can show that the formal σ-expansion of the factor $\mathcal{G}_\nu(\beta)\,[\mathcal{Z}(\sigma,1+\nu) - \mathcal{Z}(\sigma,\nu)]$ in Eq.(24) does not contain a constant term but starts with the $O(\sigma^2)$-term [4]. Here we only illustrate this fact by resorting to the formal expansion of $\mathcal{Z}(\sigma,\nu)$ in a power series of σ,

$$\mathcal{Z}(\sigma,\nu) = \frac{1}{4} + \frac{1}{12}\frac{\sigma^2}{|\nu|^2} + O(\sigma^4) \ . \qquad (26)$$

Then we observe that a particular combination $\mathcal{Z}(\sigma,1+\nu) - \mathcal{Z}(\sigma,\nu)$ cancels the common constant term $\frac{1}{4}$ in $\mathcal{Z}(\sigma,1+\nu)$ and $\mathcal{Z}(\sigma,\nu)$ and that its σ-expansion starts with the $O(\sigma^2)$-term.

Thus we can set

$$\begin{aligned} &\mathcal{Z}(\sigma,1+\nu) - \mathcal{Z}(\sigma,\nu) = \mathcal{A}(\nu)\sigma^2 + O(\sigma^4) \\ &\text{with} \\ &\mathcal{A}(\nu) := \lim_{\sigma\to 0}\frac{1}{\sigma^2}(\mathcal{Z}(\sigma,1+\nu) - \mathcal{Z}(\sigma,\nu)) \ . \end{aligned} \qquad (27)$$

Looking at Eq.(16), only the integral-region $|\nu| \lesssim \beta$ is important while it is reasonable to assume $\beta \ll 1$. Within this effective integral-region of ν ($\lesssim \beta \ll 1$), thus, it holds $\mathcal{Z}(\sigma, 1+\nu) - \mathcal{Z}(\sigma, \nu) > 0$, so that $\mathcal{A}(\nu) > 0$.

Then we reach

$$\begin{aligned}\langle \Delta v_z^2 \rangle_M / \frac{4e^2}{\pi^2 m^2 \tau^2 \sigma^2} &= \mathcal{G}_\nu(\beta) \left[\mathcal{A}(\nu)\sigma^2 + O(\sigma^4)\right] \\ &= \mathcal{A}(\beta)\sigma^2 + O(\sigma^4) \ , \end{aligned} \tag{28}$$

where $\mathcal{A}(\beta) := \mathcal{G}_\nu(\beta) \left[\mathcal{A}(\nu)\right]$.

Thus we conclude

$$\langle \Delta v_z^2 \rangle_M \approx \frac{4e^2 \mathcal{A}(\beta)}{\pi^2 m^2 \tau^2} + O\left((z/\tau)^2\right) \ . \tag{29}$$

Taking into account the quantum spread of the probe particle, thus, the observed velocity dispersion $\langle \Delta v_z^2 \rangle_M$ behaves as $1/\tau^2$ in the late-time limit. This behavior is consistent with other cases, e.g. the case with the Lorentzian switching [2].

We can now pinpoint the origin of a peculiar behavior $\langle \Delta v_z^2 \rangle_M \sim 1/z^2$ reported in Ref.[1]. There the probe particle has been assumed to be a point particle, corresponding to setting $\beta = 0$ from the outset. It is equivalent to taking the limit $\beta \to 0$ in Eq.(24) with a fixed σ. Noting Eq.(17) along with $\mathcal{Z}(\sigma, 0) = 0$, then, we see that

$$\begin{aligned}\lim_{\beta \to 0} \langle \Delta v_z^2 \rangle_M &= \frac{4e^2}{\pi^2 m^2 \tau^2 \sigma^2} \mathcal{Z}(\sigma, 1) \\ &= \frac{e^2}{4\pi^2 m^2 \tau^2 \sigma^3} \ln\left(\frac{1+\sigma}{1-\sigma}\right)^2 \ . \end{aligned} \tag{30}$$

Going back to the original variables, this result reads

$$\begin{aligned}\langle \Delta v_z^2 \rangle &= \frac{e^2 \tau}{32\pi^2 m^2 z^3} \ln\left(\frac{\tau + 2z}{\tau - 2z}\right)^2 \\ &\approx \frac{e^2}{4\pi^2 m^2 z^2} + O((z/\tau)^2) \ , \end{aligned} \tag{31}$$

exactly recovering the result shown in Ref. [1].

When one treats the probe as a point-particle, then, the factor $\mathcal{Z}(\sigma, \nu)$ in Eq.(24) *does not exist from the very beginning*, and only the factor $\mathcal{Z}(\sigma, 1+\nu)$ shows up. Then there is no cancellation of the constant term $\frac{1}{4}$ (see Eq.(26)) which would have been caused by the combination of $\mathcal{Z}(\sigma, 1+\nu) - \mathcal{Z}(\sigma, \nu)$, so that the constant term originating from $\mathcal{Z}(\sigma, 1+\nu)$ yields the peculiar behavior of $\langle \Delta v_z^2 \rangle \sim 1/z^2$ in the late-time limit. Considering Eqs.(16) and (17), we see that this phenomenon corresponds to a *measure-zero* set ($\nu = 0$) in the whole infinite ν-integral region, so that *it does not arise once the spread of the probe particle is taken into account.*

We also note that the combination $\mathcal{Z}(\sigma, 1+\nu) - \mathcal{Z}(\sigma, \nu)$ is quite similar to the *anti-correlation* effect [2] discussed in the previous section. Indeed one can compare $\mathcal{Z}(\sigma, 1+\nu) - \mathcal{Z}(\sigma, \nu)$ with the term $\langle \Delta v_z{}^2 \rangle_{MS}$ in Eq.(9) (see also Eq.(12)). Here the spatial separation between the center-part and the Gaussian tails of the probe particle is, when translated into the temporal separation in the two-point functions, is playing the similar role to the relation between the main measuring part and the switching tails in the switching function. Thus we find that the anti-correlation effect caused by the spread of the probe is essential in the measuring process of vacuum fluctuations.

V. SUMMARY AND DISCUSSIONS

In this paper, we have studied the switching effect and the smearing effect in the vacuum measurement.

Setting up a model of the electromagnetic vacuum in a half-space with an infinite, flat mirror-boundary of perfect reflectivity, we have analyzed the measurement process of the vacuum fluctuations through the velocity dispersion of a probe particle.

We have first introduced a suitable switching function and have investigated the switching effect in the measurement process. We have found out that the anti-correlation between the main measuring part and the switching tails yields an non-trivial effect in the vacuum environment. This is partially because the vacuum is essentially a zero-sum environment; the contributions from the switching tails are then dominant and cannot be negligible. Combined with the non-trivial non-local interaction between the main measuring part and the switching tails, this fact makes the switching effect quite significant in the vacuum physics.

We have next treated the probe particle as a Gaussian wave-packet and have studied the smearing effect in the measurement process of the vacuum fluctuations. We have found out that the spread of the probe particle significantly influences the measured velocity dispersion; in particular the z-component, $\langle \Delta v_z^2 \rangle$, asymptotically behaves as $\langle \Delta v_z^2 \rangle \sim 1/\tau^2$ as $\tau \to \infty$, which is consistent with other results with different models [2]. This observation resolves the reported puzzle of the peculiar late-time behavior $\langle \Delta v_z^2 \rangle \sim 1/z^2$ for a *point-particle* probe model [1]. It is now clear that the spread of the probe particle is an essential ingredient to understand the observed vacuum fluctuations. We have also noted that the formula Eq.(24) has a similar structure to the *anti-correlation* effect arisen from the interplay between the main measuring part and the switching tails (the term $\langle \Delta v_z{}^2 \rangle_{MS}$ in Eq.(9)). Indeed, the spatial separation between the center-part and the Gaussian tails of the probe particle is, when rephrased in terms of the temporal separation in the two-point functions of time, has an affinity to the relation between the main measuring part and the switching tails in the switching function. We can thus interpret Eq.(24) as the anti-correlation effect caused by the spread of the probe.

Finally, let us briefly discuss the common mechanism underlying both the switching effect and the smearing effect investigated in this paper. In the measurement process, two different time-scales are essentially involved. One is a very short time-scale, say τ_{vac}, charactering the violently fluctuating vacuum and the other is a much longer time-scale, say τ_{probe}, characterizing the size of the probe. Let us imagine taking the time-average during the time-interval τ_{probe} of a rapidly oscillating function with the period τ_{vac}. Then one expects that strong cancellations occur in the normal case $\tau_{vac} \ll \tau_{probe}$ (anti-correlation), while no effective cancellation occurs in the case $\tau_{vac} \sim \tau_{probe}$ (strong correlation). The former situation can be compared with the smooth switching case and the spread probe case, while the latter situation to the sudden switching case and the case of the point-particle probe investigated in Ref.[1].

Since the spectral profile of the vacuum can show up only through interactions with some kind of a probe (in a broad sense), the effects revealed in this paper are quite significant from not only the theoretical viewpoint but also the viewpoint of applications. It is desirable to investigate related problems along the line studied here.

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